Solutions

1. Clearly both one – one and onto
Because if n is odd, values are set of all non–negative integers and if n is an even, values are set of all negative integers.
Hence, (C) is the correct answer.

2. $z_1^2 + z_2^2 - z_1z_2 = 0$
$(z_1 + z_2)^2 - 3z_1z_2 = 0$
$a^2 = 3b$.
Hence, (C) is the correct answer.

5. $\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$
$(1 + abc)\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$
$\Rightarrow abc = -1$.
Hence, (B) is the correct answer

4. $\frac{1+i}{1-i} = \frac{(1+i)^2}{2} = i$
$\left(\frac{1+i}{1-i}\right)^x = i^x$
$\Rightarrow x = 4n$.
Hence, (A) is the correct answer.

6. Coefficient determinant = $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$
$\Rightarrow b = \frac{2ac}{a+c}$.
Hence, (C) is the correct answer

8. $x^2 - 3|x| + 2 = 0$
$(|x| - 1)(|x| - 2) = 0$
$\Rightarrow x = \pm 1, \pm 2$.
Hence, (B) is the correct answer

7. Let α, β be the roots
$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$
$\alpha + \beta = \frac{\alpha^2 + \beta^2 - 2\alpha\beta}{(\alpha+\beta)}$
$\left(-\frac{b}{a}\right) = \frac{b^2 - 2ac}{c^2}$
$\Rightarrow 2a^2c = b(a^2 + bc)$

$\Rightarrow \frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in H.P.
Hence, (C) is the correct answer

10. $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$A^2 = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$= \begin{bmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{bmatrix}$

$\Rightarrow \alpha = a^2 + b^2, \beta = 2ab.$
Hence, (B) is the correct answer.

9. $\beta = 2\alpha$

$3\alpha = \frac{3a-1}{a^2-5a+3}$

$2\alpha^2 = \frac{2}{a^2-5a+6}$

$\frac{(3a-1)^2}{a(a^2-5a+3)^2} = \frac{1}{a^2+5a+6}$

$\Rightarrow a = \frac{2}{3}.$
Hence, (A) is the correct answer

12. Clearly $5! \times 6!$
(A) is the correct answer

11. Number of choices $= {}^5C_4 \times {}^8C_6 + {}^5C_5 \times {}^8C_5$
= 140 + 56.
Hence, (B) is the correct answer

13. $\Delta = \begin{vmatrix} 1+\omega^n+\omega^{2n} & \omega^n & \omega^{2n} \\ 1+\omega^n+\omega^{2n} & \omega^{2n} & 1 \\ 1+\omega^n+\omega^{2n} & 1 & \omega^n \end{vmatrix}$

= 0
Since, $1 + \omega^n + \omega^{2n} = 0$, if n is not a multiple of 3
Therefore, the roots are identical.
Hence, (A) is the correct answer

14. ${}^nC_{r+1} + {}^nC_{r-1} + {}^nC_r + {}^nC_r$
$= {}^{n+1}C_{r+1} + {}^{n+1}C_r$
$= {}^{n+2}C_{r+1}$.
Hence, (B) is the correct answer

17. $\frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} - \ldots\ldots\ldots$

$= 1 - \frac{1}{2} - \frac{1}{2} + \frac{1}{3} + \frac{1}{3} - \frac{1}{4} - \ldots\ldots\ldots$

$= 1 - 2\left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - ....\right)$

$= 2\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...\right) - 1$

= 2 log 2 – log e

$= \log\left(\frac{4}{e}\right)$.

Hence, (D) is the correct answer.

15. General term = $^{256}C_r\,(\sqrt{3})^{256-r}\,[(5)^{1/8}]^r$

From integral terms, or should be 8k

$\Rightarrow$ k = 0 to 32.

Hence, (B) is the correct answer.

18. $f(x) = ax^2 + bx + c$

f (1) = a + b + c

f (– 1) = a – b + c

$\Rightarrow$ a + b + c = a – b + c also 2b = a + c

f′ (x) = 2ax + b = 2ax

$f'(a) = 2a^2$

f′ (b) = 2ab

f′ (c) = 2ac

$\Rightarrow$ AP.

Hence, (A) is the correct answer.

19. Result (A) is correct answer.

20. (B)

21. $a\left(\frac{1+\cos C}{2}\right) + c\left(\frac{1+\cos A}{2}\right) = \frac{3b}{2}$

$\Rightarrow$ a + c + b = 3b

a + c = 2b.

Hence, (A) is the correct answer

26. f (1) = 7

f (1 + 1) = f (1) + f (1)

f (2) = 2 × 7

only f (3) = 3 × 7

$\sum_{r=1}^{n} f(r) = 7\,(1 + 2 + \ldots\ldots\ldots + n)$

$= 7\,\frac{n(n+1)}{2}$.

25. (B)

23. $-\frac{\pi}{4} \le \frac{\sin^2 x}{2} \le \frac{\pi}{4}$

$-\frac{\pi}{4} \le \sin^{-1}(a) \le \frac{\pi}{4}$

$\frac{1}{2} \le |a| \le \frac{1}{\sqrt{2}}$.

Hence, (D) is the correct answer

27. LHS = $1 - \frac{n}{1!} + \frac{n(n-1)}{2!} - \frac{n(n-1)(n-2)}{3!} + \ldots\ldots\ldots$

$= 1 - {}^nC_1 + {}^nC_2 - \ldots\ldots\ldots$

$= 0$.

Hence, (C) is the correct answer

30. $\lim\limits_{x\to 0} \frac{\frac{1}{3+x} + \frac{1}{3-x}}{1} = \frac{2}{3}$.

Hence, (C) is the correct answer.

28. $4 - x^2 \neq 0$

$\Rightarrow x \neq \pm 2$

$x^3 - x > 0$

$\Rightarrow x\,(x + 1)\,(x - 1) > 0$.

Hence (D) is the correct answer.

29. $\lim\limits_{x\to\pi/2} \frac{\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)(1 - \sin x)}{4\left(\frac{\pi}{4} - \frac{x}{2}\right)(\pi - 2x)^2}$

$= \frac{1}{32}$.

Hence, (C) is the correct answer.

32. $f(-x) = -f(x)$

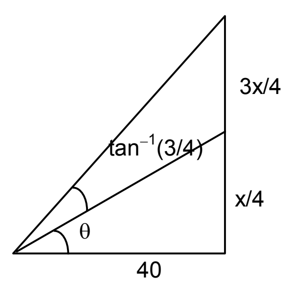
Hence, (B) is the correct answer.

1. $\sin(\theta + \alpha) = \frac{x}{40}$

$\sin a = \frac{x}{140}$

$\Rightarrow x = 40$.

Hence, (B) is the correct answer



34. $f(x) = 0$ at $x = p, q$

$6p^2 + 18ap + 12a^2 = 0$

$6q^2 + 18aq + 12a^2 = 0$

$f''(x) < 0$ at $x = p$

and $f''(x) > 0$ at $x = q$.

30. Applying L. Hospital's Rule

$\lim\limits_{x\to 2a} \frac{f(a)g'(a) - g(a)f'(a)}{g'(a) - f'(a)} = 4$

$$\frac{k(g'(a) - ff'(a))}{(g'(a) - f'(a))} = 4$$

k = 4.

Hence, (A) is the correct answer.

36. $\int_a^b x\, f(x)\, dx$

$= \int_a^b (a + b - x)\ f(a + b - x)\, dx.$

Hence, (B) is the correct answer.

33. f′ (0)

f′ (0 – h) = 1

f′ (0 + h) = 0

LHD ≠ RHD.

Hence, (B) is the correct answer.

37. $\lim_{x\to 0} \frac{\tan(x^2)}{x \sin x}$

$= \lim_{x\to 0} \frac{\tan(x^2)}{x^2\left(\frac{\sin x}{x}\right)}$

= 1.

Hence (C) is the correct answer.

38. $\int_0^1 x\,(1 - x)^n\, dx = \int_0^1 x^n\,(1 - x)$

$= \int_0^1 (x^n - x^{n+1}) = \frac{1}{n+1} - \frac{1}{n+2}.$

Hence, (C) is the correct answer.

35. $F(t) = \int_0^t f(t - y)\, f(y)\, dy$

$= \int_0^t f(y)\, f(t - y)\, dy$

$= \int_0^t e^y\,(t - y)\, dy$

$= x^t - (1 + t).$

Hence, (B) is the correct answer.

34. Clearly f″ (x) > 0 for x = 2a ⇒ q = 2a < 0 for x = a ⇒ p = a

or $p^2 = q \Rightarrow a = 2$.

Hence, (C) is the correct answer.

40. $F'(x) = \frac{e^{\sin x}}{3^x}$

$= \int \frac{3}{x} e^{\sin x}\, dx = F(k) - F(1)$

$= \int_1^{64} \frac{e^{\sin x}}{x}\, dx = F(k) - F(1)$

$= \int_1^{64} F'(x)\, dx = F(k) - F(1)$

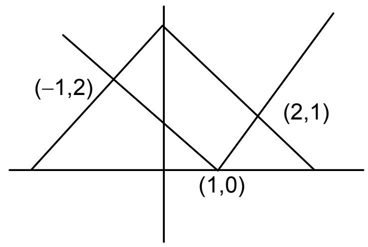
$F(64) - F(1) = F(k) - F(1)$

$\Rightarrow k = 64.$

Hence, (D) is the correct answer.

41. Clearly area = $2\sqrt{2} \times \sqrt{2}$

= sq units



45. Let p (x, y)

$(x - a_1)^2 + (y - b_1)^2 = (x - a_2)^2 + (y - b_2)^2$

$(a_1 - a_2)\, x + (b_1 - b_2)\, y + \frac{1}{2}(b_2^2 - b_1^2 + a_2^2 - a_1^2) = 0.$

Hence, (A) is the correct answer.

46. $x = \frac{a\cos t + b\sin t + 1}{3}$, $y = \frac{a\sin t - b\cos t + 1}{3}$

$\left(x - \frac{1}{3}\right)^2 + y^2 = \frac{a^2 + b^2}{9}.$

Hence, (B) is the correct answer.

43. Equation $y^2 = 4a\ 9x - h)$

$2yy_1 = 4a \Rightarrow yy_1 = 2a$

$yy_2 = y_1^2 = 0.$

Hence (B) is the correct answer.

42. $\int_0^1 f(x)\, [x^2 - f(x)]\, dx$

solving this by putting $f'(x) = f(x)$.

Hence, (B) is the correct answer.

50. Intersection of diameter is the point (1, – 1)

$\pi s^2 = 154$

$\Rightarrow s^2 = 49$

$(x - 1)^2 + (y + 1)^2 = 49$

Hence, (C) is the correct answer.

47. (D)

49. $\frac{dx}{dy}\,(1 + y^2) = (e^{\sin^{-1} y} - x)$

$$\frac{dx}{dy} + \frac{x}{1+y^{\alpha}} = \frac{e^{sub^{-1}-y}}{1+y^2}$$

52. $$\frac{x^2}{\left(\frac{12}{5}\right)^2} - \frac{y^2}{\left(\frac{9}{5}\right)^2} = 1$$

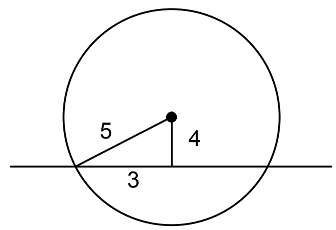
$\Rightarrow e_1 = \frac{5}{4}$

$ae_2 = \sqrt{1-\frac{b^2}{16}} \times 4 = 3$

$\Rightarrow b^2 = 7.$

Hence, (C) is the correct answer.

54. (C)



69. np = 4

npq = 2

$q = \frac{1}{2}, p = \frac{1}{2}$

n = 8

$p\ (x = 1) = {}^8C_1 \left(\frac{1}{2}\right)^8$

$= \frac{1}{32}.$

Hence, (A) is the correct answer.

49. $(x-1)^2 + (y-3)^2 = r^2$

$(x-4)^2 + (y+2)^2 - 16 - 4 + 8 = 0$

$(x-4)^2 + (y+2)^2 = 12.$

67. Select 2 out of 5

$= \frac{2}{5}.$

Hence, (D) is the correct answer.

65. $0 \le \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1$

$12x + 4 + 3 - 3x + 6 - 12x \le 1$

$0 \le 13 - 3x \le 12$

$3x \le 13$

$\Rightarrow x \ge \frac{1}{3}$

$x \le \frac{13}{3}.$

Hence, (C) is the correct answer.

3. $\text{Arg}\left(\frac{z}{\omega}\right) = \frac{\pi}{2}$

$|z\omega| = 1$

$\bar{z}\,\omega = -i$ or $+i$.